# SLOPING SURCHARGES

ANALYTICAL TRIAL WEDGE FOR SHORING SYSTEMS
UTILIZING COHESION IN COMBINATION WITH GRANULAR SOIL

An alternative to the semi-graphical trial wedge is the analytical trial and error approach. The analytical method involves equating the forces acting on a straight line failure plane, at an assumed failure angle (a), then solving for the maximum resisting active resultant pressure  $(P_h)$ .

This analytical method will be useful for irregular slopes and for slopes where Rankine, Coulomb or Log-spiral applications are not appropriate. In addition, when the soil contains clay the additional clay shear resistance acting along the. active failure plane surface effectively reduces the active wedge driving force.

The analytical procedure consists of equating forces, assuming a failure angle, then computing a resulting pressure. Computation is repeated until the failure wedge angle selected produces the maximum resulting active pressure. The analytical procedure is ideal for computer application.

The maximum resulting active pressure can be used to determine an equivalent fluid pressure (Kw) which would simulate a soil having a level surface acting on the height of the shoring system. From the equivalent fluid (Kw) value a fictitious  $K_{\mathtt{A}}$  value may be computed. The method for converting the maximum resulting pressure to Kw and then to a hypothetical  $K_{\mathtt{A}}$  times the soil unit weight is demonstrated in Part 2.

Active wedge movement or clay shrinkage will permit the development of clay tension cracks. Soil can lose apparent cohesion, due to loss of capillary tension for example. The tension cracks will eventually extend to the wedge failure plane with the maximum crack height equaling the critical height of the clay.

When tension cracks can develop, it will only be necessary to consider the soil wedge weight, including any surcharge, between the limits of the shoring and the nearest tension crack that intercepts the failure plane. Otherwise, the entire wedge soil weight, including surcharge, should be deemed to be acting as the driving force on the failure plane.

Cohesive resistance acting along the active failure plane should only be considered to be effective between the limits of the shoring and any tension crack that intercepts the failure plane.

TRENCHING AND SHORING MEMO NO.1 (10/90)

If tension cracking will not occur, the entire length of the failure plane will provide cohesive resistance. Limiting the length of the clay resistance along the hypothetical failure line is demonstrated in Part 3 and Part 4.

When it is possible for water to stand in tension cracks the additional horizontal reaction must be added to the other force components. The horizontal component of water ( $P_u = 0.5\gamma_u h_c^2$ ) will act at one-third the height of the crack and will oppose the horizontal component of the shoring resisting force ( $P_h$ ).

When soilshear strength has both frictional and cohesive components these values must be confirmed and be appropriate. Log-of-Test Borings accompanying contract plans do not indicate both cohesion and friction angle for the same boring. It will generally be necessary for the Contractor to furnish soils information from a recognized soils lab or similar source.

Verification of soil shear strength with values derived from the Log-of-Test Borings should be based on a conservative interpretation of the Log-of-Test Borings and the relationships between Standard Penetration Test (SPT) N-values and soil shear strength values. Generally speaking, unconfined compressive strength derived from SPT N-values for a cohesive soilshould be based on a more conservative interpretation than that used to derive a soil friction angle from SPT N-value for a granular soil. This also means that when TABLES 12 and 13 are used conservative values should be selected. Conformance with the foregoing will obviate the need to apply safety factors to the soil strength values.

Wall friction (6) may be considered when the shoring will not be subjected to dynamic loading, or when shoring construction does not include lagging. For the active condition the wall friction acts vertically upward and effectively reduces the force of the weight acting down. It is recommended that no wall friction be used for the passive case.

It should be noted that weak seams in the soil can predispose the soil mass to fail along the seam rather than along the failure plane, particularly if water is present in the seam. Weak seam slippage would prevent the full wedge from becoming active. If weak seams exist in the passive wedge undesirable events will most probably result.

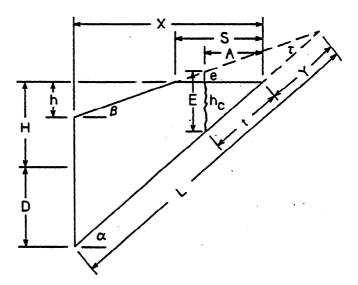
The analytical development for trial wedges assumes homogeneous isotropic soil. Surcharges on the active wedge merely add weight to the wedge, and with the analytical method any surcharges situated beyond the failure plane, or behind tension cracks, are neglected.

Analytical trial wedge development is presented in the following sequence:

- PART 1: Effect of tension crack height and location on the active trial wedge.
- PART 2: Analytic&l active-trial wedge for strutted type systems assuming no clay tension crack has developed in the wedge. An example problem is included.
- PART 3: Analytical active trial wedge development for a continuous cantilever shoring wall with a surcharge situated over a clay tension crack.
- PART 4: Example problem demonstrating passive resistance for the active wedge of PART 3 for a level and for a downward sloping surface. Recommended safety factors against sliding and overturning are included.

# PART 1: EFFECT OF TENSION CRACK ON THE ACTIVE WEDGE

Three different conditions can exist for the location of the maximum potential clay tension crack: (1) it can be located within the semi-level top of embankment area as shown in the adjacent figure where S > A, (2) it can be located below the hinge point so that A >S, or (3) it may be located on a slope (S = 0) when the slope line intersects the failure plane. With no surcharge on the embankment the essential equations for the length



LOCATION OF TENSION CRACK

- (t) and for the contributing weight (W) are as follows:
- 1) S > A

 $S = X - h/tan\beta$ 

 $Y = S(sin\beta)(sin\tau)$ 

 $A = h_c/\tan\alpha = \tan\alpha$ 

$$t = h_c/\sin\alpha = A/\cos\alpha$$
  
 $E = (t + Y)(\sin\tau)/\sin(90^{\circ} + \beta)$   
 $e = E - h_c$   
 $S - A = e/\tan\beta$   
 $W = \gamma/2[(H + D - h + E)(X - A) - e^2/\tan\beta]$ 

$$S = X - h/\tan\beta$$

$$t = [h_c \sin(90^\circ + \beta) - S(\sin\beta)]/\sin\beta$$

$$A = t\cos\alpha$$

$$W = \gamma/2[(H + D - h + h_c)(X - A)]$$

3) 8 = 0

$$t = h_c \sin(90^\circ + \beta)/\sin \tau$$

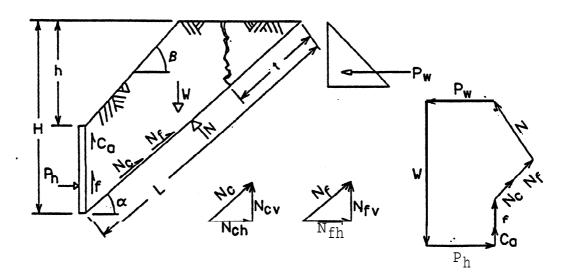
$$A = t\cos \alpha$$

$$W = \gamma/2[(H + D - h + h_c)(X - A)]$$

When a surcharge of a permanent nature (not traffic) is situated on the embankment the depth of the tension crack  $(h_c)$  which might. be located under the surcharge (Q) is reduced as indicated by the following equation:

Under surcharge (Q): 
$$h_c = (2C/[K_A]^{1/2} - Q)/\gamma$$

## ANALYTICAL ACTIVE TRIAL WEDGE FOR STRUTTED TYPE SYSTEMS



TRENCHING AND SHORING MEMO NO.1 (10/90)

The following is an analytical development of an active wedge for a strutted type system with an intermix of cohesionless and cohesive soils. Assume insufficient time for a significant clay tension crack to develop. Assume also that no water will get into tension cracks and that no friction or other cohesive forces will develop on the soil face of the shoring. An example problem is included.

 $N_f$  = Frictional resistance  $\gamma$  = Unit weight of the soil (Granular)

 $N_c$  = Cohesive resistance

 $\phi$  = Soil internal friction angle

C = Clay cohesion

 $\alpha$  = Assumed angle of wedge failure

W = Wedge weight plus surcharge

Shear on failure plane:  $S = C + Ntan \phi = N_C + N_f$ 

The relatively small wall adhesion (CJ forces will not be considered to simplify the calculations. Wall friction  $(\delta)$  and water pressure  $(\textbf{P}_{\textbf{W}})$  are not included in this example.

 $N_f = N tan \phi$ 

 $N_{fH} = Ntan\phi cos\alpha$ 

 $N_{tv} = Ntan\phisin\alpha$ 

 $N_c = LC = HC/sin\alpha$ 

 $N_{cH} = N_{c}\cos\alpha = HC(\cos\alpha)/\sin\alpha$ 

 $N_{CH} = HC/tan\alpha$ 

 $N_{rv} = N_c \sin \alpha = HC(\sin \alpha)/\sin \alpha$ 

 $N_{cv} = HC$ 

EQUATE  $\Sigma F_H = 0 \rightarrow +$ 

 $\begin{array}{lll} 0 &=& P_h &+& N_{CH} &+& N_{fH} &-& N sin\alpha \\ 0 &=& P_h &+& HC/tan\alpha &+& N tan\phi cos\alpha &-& N sin\alpha \\ 0 &=& P_h &+& HC/tan\alpha &+& N (tan\phi cos\alpha &-& sin\alpha) \end{array}$ 

EQUATE  $\Sigma F_v = 0$ 

 $0 = -W + N_{CV} + N_{fV} + N_{COS\alpha}$  0 = W - HC - Ntanφsinα - Ncosα

 $N = (W - HC)/(tan\phi sin\alpha + cos\alpha)$ 

Or transposing:  $N = (W - HC)\cos\phi/\cos(\alpha - \phi)$ 

From  $\Sigma F_n = 0$ :

 $P_h = N(\sin\alpha - \tan\phi\cos\alpha) - HC/\tan\alpha$ 

Substitute the N value from the  $\Sigma F_v = 0$  equation

 $P_{h} = [(W - HC)/(\tan\phi\sin\alpha + \cos\alpha)][\sin\alpha - \tan\phi\cos\alpha] - HC/\tan\alpha$   $P_{h} = (\sin\alpha - \tan\phi\cos\alpha)(W - HC)/(\tan\phi\sin\alpha + \cos\alpha) - HC/\tan\alpha$   $Transposing: P_{h} = (W - HC)\tan(\alpha - \phi) - HC/\tan\alpha$ 

If C = 0 then  $P_h = W \tan (\alpha - \phi)$ 

Where  $W = \gamma/2(H^2/\tan \alpha - h^2/\tan \beta)$ , and  $\beta \le \phi$ 

Note: If there is no bench on the embankment portion of the wedge the equations above are still valid.

Assume: (Generally  $45^{\circ}-\phi/2 < \alpha < 45^{\circ}+\phi/2$ )

Search values of  $\alpha$  to determine maximum  $P_h$ .

Once maximum  $P_h$  is determined an equivalent fluid weight may be calculated from the equation  $Kw = 2P_h(H-h)^2$ .

If the system will be in place long enough for clay tension cracks to develop, the clay resisting length (L) must be reduced by the length (t) along the failure plane behind the tension crack. The height of the tension crack  $h_{\rm c}$ , determined by the critical height of the clay, is equal to  $(2{\rm C}/[{\rm K_A}]^{1/2}$  - Q)/ $\gamma$ , where Q is the surcharge and  ${\rm K_A}$  is for a level surface. The length t may be determined from the triangular relationships  $t/h_{\rm c}$  = L/H. Water pressure may now need to be considered in addition to all other forces acting on the system.

### EXAMPLE:

Assume foregoing configuration for a strutted trench with H equal to 28 feet, h = 16 feet (which includes 2 feet of soil for an equivalent minimum surcharge),  $\beta = 45^{\circ}$ ,  $\gamma = 115$  pcf,  $\phi = 32^{\circ}$ , with  $\delta = 0$ , and C = 200 psf. This condition is for short term (48 hours) loading, and appropriate safety factors are included in the soil parameters.

Determine maximum active resultant pressure  $(\mathtt{P}_h)$  and an equivalent fluid weight (Kw). From Kw determine a hypothetical  $\mathtt{K}_\mathtt{A}$  using the given unit weight. Draw a proper pressure diagram for the given height of the shoring system.

### SOLUTION:

For short term loading the potential clay cracking will be ignored.

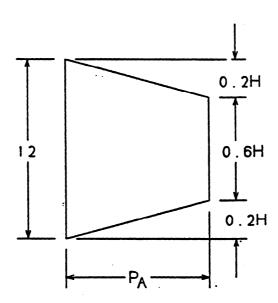
**Tan**
$$\phi$$
 tan32° = 0.625 **tan** $\beta$  = tan45° = 1.0  
W = 115/2[(28)<sup>2</sup>/ **tan** $\alpha$  - (16)<sup>2</sup>/1.00] = 57.5[784/ **tan** $\alpha$  - 256)  
HC = 28(200) = 5,600

# $P_h = (\sin\alpha - 0.625\cos\alpha)[57.5(784/\tan\alpha - 256) - 5,600]/(0.625\sin\alpha + \cos\alpha) - HC/\tan\alpha$

Successively try various angles in the above equation to search for the maximum active pressure:

Maximum 
$$P_h$$
 = 1,048 Lb/LF (When angle  $\alpha$  = 52°)   
 Kw = 2P/(H - h)² = 2(1048)/(28-16)² 15 pcf   
 Then a hypothetical  $K_A$  = **Kw/y** = 15/115 = 0.13

And one appropriate pressure diagram to use is the trapezoid for which  $P_A$  = 0.8Kw(H - h) = 0.8(15)(12) = 144 psf



If C = 0 the exposed slope angle cannot be assumed to be greater than the internal friction angle of the soil  $(\beta \leq \phi)$ 

Revising so that C = 0 and  $\beta = \phi$ :

TRENCHING AND SHORING MEMO NO.1 (10/90)

W = 
$$115/2[(28)^2/$$
 'tan $\alpha$ -  $(16)^2/$  tan $\phi$ ]

Maximum P<sub>h</sub> = Wtan( $\alpha$  -  $\phi$ ) = 4981 Lb/L.F. (When  $\alpha$  = 46°)

Then Kw =  $2(4981)/(28-16)^2$  = 69 pcf

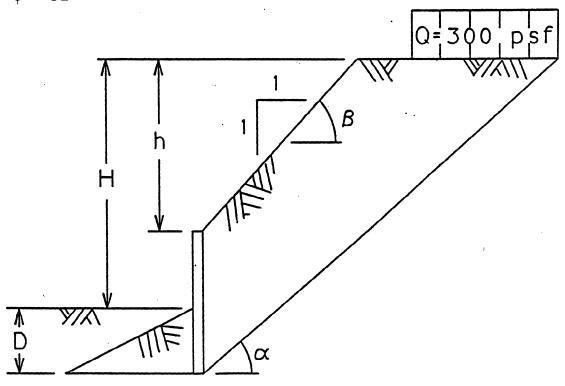
The hyothetical K<sub>A</sub> =**Kw/** $\gamma$  = 69/115 = 0.6

and P<sub>A</sub> = 0.8Kw(H - h) = 0.8(69)(12) = 662 psf > 144 psf

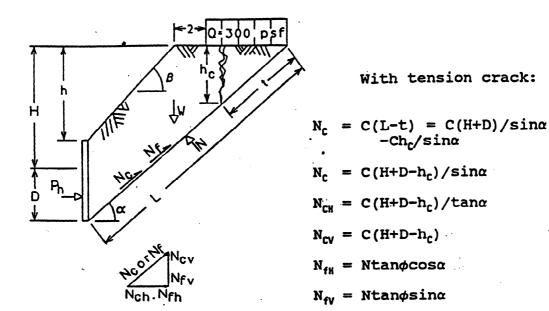
## PART 3. ANALYTICAL WEDGE FOR A CANTILEVER WALL

Following is a development of an analytical active wedge for a continuous cantilever type shoring wall with a surcharge and a clay tension crack using the following parameters:

$$\begin{array}{ll} H = 24 \\ h = 12 \end{array}$$
 
$$\begin{array}{ll} \beta = 45 \\ \delta = 0 \end{array}$$
 
$$\begin{array}{ll} D = 20 \\ \gamma = 115 \\ \phi = 32 \end{array}$$
 
$$\begin{array}{ll} C = 200 \text{ psf} \\ \text{Surcharge} = 300 \text{ psf} \end{array}$$



## ANALYTICAL ACTIVE WEDGE



A) For the case when a tension crack will develop:

$$\Sigma F_{H} = 0$$

$$0 = P_{H} + N_{CH} + N_{fH} - Nsin\alpha$$

$$0 = P_{h} + C(H+D-h_{c})/tan\alpha + Ntan\phicos\alpha - Nsin\alpha$$

$$P_{h} = N(sin\alpha - tan\phicos\alpha) - C(H+D-h_{c})/tan\alpha$$

Transposing:  $P_h = N\sin(\alpha - \phi)/\cos\phi - C(H + D - h_c)/\tan\alpha$ 

$$\Sigma F_{V} = 0$$

$$0 = N_{cv} + N_{fv} + N_{cos\alpha} - W$$
 (Where W includes Q)

$$0 = C(H+D-h_c)$$
 -Ntanøsina + Ncosa - W

Solve for N and substitute into the  $\Sigma F_{H} = 0$  equation.

$$N = [W - C (H+D-h_c)]/(tan\phisin\alpha + cos\alpha)$$

Transposing:  $N = [W - C(H + D - h_c)/(\cos (\alpha - \phi)/\cos\phi]$ 

$$p_{h} = \left\{ \begin{bmatrix} W - C(H+D-h_{c}) \end{bmatrix} / (tan\phisin\alpha + cos\alpha) \right\} \left\{ sin\alpha - tan\phicos\alpha \right\}$$

 $P_{h} = [\sin\alpha - \tan\phi\cos\alpha] \{ [W-C(H+D-h_{c})] / (\tan\phi\sin\alpha + \cos\alpha) \} - C(H+D-h_{c}) / \tan\alpha$ 

Transposing:  $P_h = [W - C(H+D-h_c)] \tan(\alpha - \phi) - C(H+D-h_c/\tan\alpha)$ 

 $L = (H+D)/\sin \alpha$ 

Surcharge between shoring and tension crack:

Partial Surcharge =  $300\{[(H+D)/\tan\alpha] - h - 2' - h_c/\tan\alpha\}$ 

 $W = \gamma/2[(H + D)^2/\tan\alpha - h^2/\tan\beta - (h_c)^2/\tan\alpha] \text{ for the soil plus } 300[(H+D-h_c)/\tan\alpha - h - 2'] \text{ for the surcharge.}$ 

 $h_c = (2C/[K_A]^{1/2} - Q)/\gamma = (400/[0.31)^{1/2} - 300)/115 = 3.64'$  (Reminder: tension crack  $K_A$  is for level surface.)

 $P_h = 22,321 \text{ Lb/LF} \text{ (maximum when } \alpha = 57^{\circ}\text{)}$ 

Confirm that the tension crack is under the surcharge; if it is not, recompute  $h_c$  (using Q = 0), W, and  $P_h$ . In this example the tension crack is under the surcharge.

B) If water might fill the tension crack the transposed  $P_{\rm h}$  equation becomes:

$$P_{h} = [W - C(H + D - h_{c})] \tan(\alpha - \phi) - C(H + D - h_{c}) / \tan\alpha + 0.5\gamma_{u}(h_{c})^{2}$$

For this condition  $P_A$  = 22,734 Lb/LF (Maximum when  $\alpha$  = 57°)

c If there will be no tension crack:

 $W = \gamma/2[(H + D)^2/\tan\alpha - h^2/\tan\beta] \text{ for the soil}$ plus 300{[(H + D)/\tan\alpha] - h - 2'} for the surcharge

$$P_h = [W - C(H + D)] \tan(\alpha - \phi) - C(H + D) / \tan\alpha$$

$$P_h$$
 = 22,072 Lb/L.F. (Maximum when  $\alpha$  = 58°)

The above equations can be used when C = 0, but in that case the slope angle (  $\beta$  ) cannot exceed the interior friction angle (  $\phi$  ) of the soil.

Use the maximum active condition (B) to determine safety factors for a passive condition where the slope is not steep but may be continuous, and compare to the condition where the excavated surface may be considered level. This example is demonstrated after the passive wedge development which follows.

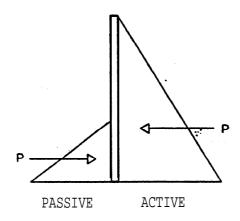
# PART 4: PASSIVE WEDGE RESISTANCE

Demonstrate passive wedge resistance for two conditions; one in which the resisting ground surface is level in front of the shoring, and one in which the ground in front of the shoring slopes down and away on approximately a 3 to 1 continuous slope. The appropriate passive wedge resisting pressure is determined from the log-spiral curves of FIGURE 8. A semigraphical or analytical passive trial wedge solution would not be appropriate since either of these two methods would approximate the Coulomb solution; which has been demonstrated to always be an unsafe approach.

### SOLUTION:

	FOR LEVEL SURFACE	SLOPING AT ≈ 3:1
β/φ	0	-18.44°/32° ≈ -0.6
$\mathbf{K}_{\!oldsymbol{ ho}}$ Coefficient	7.9	3.2
Reduction Fact	or 0.425	0.425
K <sub>p</sub>	3.36	1.36
D	20'	20'
γ	115 pcf	115 pcf
$P_p = 1/2K_p\gamma D^2$	77,280 Lb/LF	31,280 Lb/LF

Wall pressures for cantilever systems are normally assumed to act at one-third of the height of a triangle (whose base is located at the bottom of the pressure wedge).



## SUMMARY OF PRESSURES:

ACTIVE PRESSURES (Maximum):

With tension crack:  $P_A$  = 22,321 Lb/LF With water in the crack:  $P_A$  = 22,734 Lb/LF (Controls) With no tension crack:  $P_A$  = 22,072 Lb/LF

PASSIVE PRESSURES (Minimum):

Sloping embankment below H:  $P_p = 31,280 \text{ Lb/LF}$  With level surface below H:  $P_p = 77,280 \text{ Lb/LF}$ 

Compute the safety factor (S.F.) against sliding:

Passive sloping embankment condition:

S.F. = 
$$P_p/P_A$$
 = 31,280/22,734 = 1.38 < 2.0 : n.g.

Passive level surface condition:

Compute the safety factor (S.F.) against overturning using the moments taken about the bottom of the shoring:

Passive sloping embankment condition:

$$M_p$$
 for  $P_p = 31,280(20)/3 = 208,533 Ft-Lb/LF$ 

$$M_A$$
 for  $P_A = 22,734(12 + 20)/3 = 242,496 Ft-Lb/LF$ 

S.F. = 
$$M_p/M_A$$
 = 208,533/242,496 = 0.86 < 1.5 : n.g.

Passive level surface condition:

$$M_p$$
 for  $P_p = 77,280(20)/3 = 515,200 Ft-Lb/LF$ 

$$M_{\lambda}$$
 for  $P_{\lambda} = 242,496$  Ft-Lb/LF

S.F. = 
$$M_p/M_A$$
 = 515,200/242,496 = 2.12 > 2.0 : o.k.

The shoring should be satisfactory for the condition where the passive wedge consists of a level surface: providing the ground conditions do not change significantly, including the possibility of the active wedge becoming heavily saturated.